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Is There a Linear Subspace in Which the Difference Vectors of Word Analogy Pairs are Parallel?

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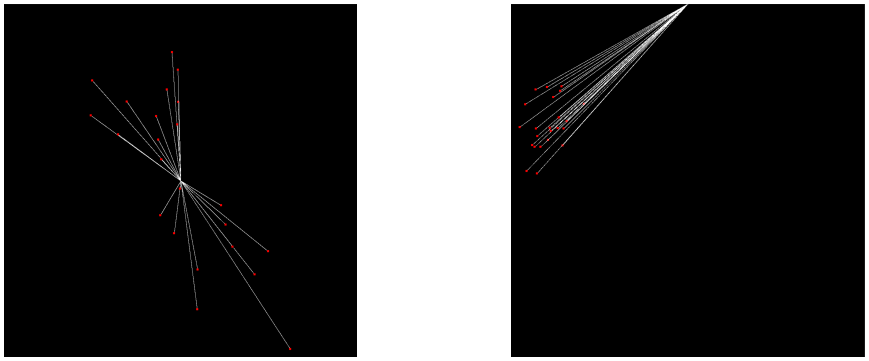
Abstract. Since Mikolov introduced word analogies as an example of semantic composition by vector addition, they have inspired both enthusiasm and disdain. If the arithmetic computation works, the relationship encoded in the word vectors should manifest itself as parallel difference vectors, and if the difference vectors are parallel, this should appear in two-dimensional projections. For Principal Component Analysis (PCA) bases computed on just the words of a relation's pairs, this seems to be true. However, PCA on larger subsets of the vocabulary typically shows a wide range of directions for difference vectors in the same relation. The PCA phenomenon is evidence for our suggestion that there is a subspace for each relation, in which the difference vectors are parallel. That is, only a subset of the semantic information for each word participates in the relation. To approximate such a subspace, we train a linear transformation which moves a portion of the pairs in a relation so that the difference vectors are nearly parallel to each other, while minimizing the movement of unrelated words. We see that there is a net improvement in evaluating not only analogies which include pairs in the training set, but also analogies between held-out pairs in the same relation. The trained transformation thus seems to isolate semantic components expressed by the relation.

Keywords: Word analogies, word vector semantics space, semantic composition.

1 Introduction

Mikolov et al. [11] introduced solving word analogies with vector arithmetic in the same paper that introduced **word2vec**, the algorithms and software release for rapidly constructing CBOW and Skip-gram semantic vector spaces. Word analogies fired the imagination, because they are similar to some standard questions on human intelligence tests [17]. Briefly, Mikolov [11] asserted that if $\mathbf{s}(word)$ is the vector for $word$, then the word analogy *Man:King :: Woman:?* can be solved for *?* = *Queen* by finding the word closest in the vector space to $\mathbf{s}(\textit{King}) - \mathbf{s}(\textit{man}) + \mathbf{s}(\textit{woman})$. They provided a corpus of word analogies which test this idea, now called the Google word analogy corpus, which we use in this paper.



(a) PCA uses whole vocabulary **(b)** PCA uses only relation file

Fig. 1. Various 2-D projections of `capital-common-countries`, using PCA on different sets of vectors.

This computation scheme is equivalent to the claim that in the semantic vector space, vector addition is equivalent to semantic combination, a claim made later that same year by Mikolov [12].

In Figure 1, we show two different PCA projections of the difference vectors for the capital-common-countries relation into a two-dimensional vector space. The difference vectors are all aligned to start from the origin, which brings out the parallelism, or lack of it, sharply. Because each figure has a different set of basis vectors, each rosette is aligned and centered differently. The difference vectors are closest to parallel in Figure 1B, in which the eigenvectors, or axes of the projection, are computed based only on the 23 distinct pairs, or 46 words, in the relation.

In general, the difference vectors in analogy files (hereafter *relations*) are not parallel in 300 dimensions [9,15] but can look more aligned after projection to two dimensions, particularly if the PCA is done on a subset of the data. If their difference vectors were actually parallel and of the same length, then word analogies would have 100% evaluation success, which would make them formidable tools, instead of the interesting demonstrations which they now are.

Although none of the PCA projections captures much of the variation, they make it seem plausible that there might be a linear transformation of the vector space which minimizes information irrelevant to a particular relation. We set ourselves the goal of finding, for each relation, a linear transformation such that the difference vectors in the transformed space would be parallel.

2 Related Work

Coecke et al. [1] proposed building an algebra for composing word vectors into phrases. Mitchell and Lapata [14] consider how vectors in a semantic space, in their case Latent Semantic Analysis (LSA) vectors describing usage context for individual words, might be composed into phrases.

They asked human beings to rate phrase similarity for two-word phrases, and attempted to correlate the human similarity scores with scores computed using nine different *composition functions* applied to the LSA vectors of the individual words. They found the best correlation for element-wise multiplication. Vector addition, as proposed by Mikolov et al. [11,12] was in the middle of the group of tested composition functions. Levy et al. [7] make an early investigation of vector-space word analogies; their article includes two versions of the vector-addition [13] method, and a new scheme of their own.

They exhibit a very-high dimension word embedding, in which context counts are dimensions, to explain how vector addition could plausibly perform semantic composition. They describe all the algorithms as forms of vector similarity calculation. In [8] Levy examines various examples of hypernym-analogies, and concludes that what they measure is not whether the analogy is valid, but whether or not a word is a hypernym of *something*. Linzen [9] argues that frequently the difference vector is irrelevant in analogy evaluation, and that a nearest neighbor effect overwhelms it.

Drozd et al. [2] notes that averaging the difference vector over many pairs makes it more reliable, and that then using logistic regression to determine a class for second words in a pair for the relation and checking whether a proposed answer is in the class gives excellent results. They tested their algorithm using two held-out pairs in the relation. This work has more emphasis on 'solving' analogies than ours, but does consider smoothing difference vectors to be worthwhile.

Finley et al. [3] test whether the vector-arithmetic strategy (without exclusion) is an improvement on word similarity on several test-sets. They concluded that some relations don't work well, but several do; they categorize the most successful analogy types as *Inflectional Morphology*; *Named Entities*; *Gendered Nouns*. Least successful types are *Derivational Morphology* and *Lexical Semantics*.

Gittens et al. [4] consider semantic combination, as a mathematical operation on word vectors in a syntactic space generated with the Skip-gram algorithm.

They provide a model for paraphrases in terms of the target and context vectors used in Skip-Gram algorithm, which leads to a formula for finding them, and conclude that when Skip-gram is applied to a corpus with a uniform word distribution, semantic combination is vector addition.

Natural language words follow a Zipf-like distribution, but Finley [3] note that analogies in which all four words have similar frequency seem to work better.

Vylomova et al. [18] train linear kernel SVMs to determine from their difference vectors whether pairs were members of a relation or not.

They find using negative samples in the training set improves precision but lowers recall. Konkol et al. [6] trained a linear transform from place-names to geographical co-ordinates, demonstrating that vector semantic spaces apparently have some location information encoded in them.

Szymanski [16] obtained a similar result with a different method, by training word vectors with parallel corpora from specific time periods in order to look for equivalents such as Ronald Reagan in 1987 is like Bill Clinton in 1997.

Table 1. Some statistics on the Google word analogies.

relation	analogies	pairs	base correct	base spares	base accuracy
capital-common-countries	506	23	179	236	0.35
capital-world	1482	39	380	753	0.26
city-in-state	1560	40	304	942	0.19
country-currency	342	19	45	109	0.13
family	506	23	177	253	0.35
gram1-adjective-adverb	992	32	17	271	0.02
gram2-opposite	756	28	15	300	0.02
gram3-comparative	1332	37	501	687	0.38
gram4-superlative	1122	34	234	699	0.21
gram5-present-participle	1056	33	64	771	0.06
gram6-nationality-adjective	1482	39	1166	203	0.79
gram7-past-tense	1560	40	161	871	0.10
gram8-plural	1332	37	67	1153	0.05
gram9-plural-verbs	870	30	136	462	0.16

3 Methodology

3.1 Public Data

We downloaded a semantic vector space file[3] and edited it to keep only the first 150,000 words.The vectors for this file have 300 32-bit floating point elements. We edited the Google analogy dataset[4] to break the analogies into pairs, which lets us consider training- and test-sets from among the pairs in each analogy type, which we then call a *relation*. The fourteen relations of the Google analogy set, and a few statistics about each are shown in Table 1.

Some of the relations have fewer pairs than appear in the downloaded test set; this is because pairs with out-of-vocabulary words are eliminated. This occurs because we limited the size of the vocabulary to 150 thousand words. These are the most frequent words in the Google news corpus from which the word vector space was built.

The downloaded file has vectors for about 3 million word-forms, but searching through so many vectors to find a near-match takes 20 times as long as searching through the smaller list. Also, using a restricted vocabulary removes some competition for the target word, perhaps resulting in higher reported accuracy. The **analogies** column in Table 1 is computed from the number of pairs as:

$$\text{analogies} = \text{pairs}(\text{pairs} - 1). \tag{1}$$

Thus for any two different pairs, we can make two analogies. For some analogy types we could make four by reversing the pairs, but we do not do this. For example the analogy (from the `gram8-plural` relation):

$$\text{mouse} : \text{mice} :: \text{cat} : \text{cats}. \tag{2}$$

[3] GoogleNews-vectors-negative300.bin.gz from https://code.google.com/archive/p/word2vec/

[4] http://download.tensorflow.org/data/questions-words.txt

could be manipulated to put any of the four words in the final position that the algorithm calculates. However, this is not obviously true for the analogy (from the `city-in-state` relation):

$$\text{Fresno : California :: Tucson : Arizona.} \tag{3}$$

because if we provide the state, there seem to be many equally good answers for a city in the state. The **base correct** column is the number of analogies for which vector addition provides the expected answer as the first choice. Mikolov's version of the algorithm excludes the three other words in the analogy from being considered.

We provide the column **spares** to record when the correct answer would be provided by this work-around. Although word similarity is interesting, it has nothing to do with phrase composition by vector addition. A glance at the table shows that spares are a major contributor to the usual statistics for these relations.

3.2 Transforming the Semantic Space

The semantic vector space $\mathbf{S}$ is defined by a function, where W is a set of words:

$$\mathbf{s}(w) : W \to \mathbb{R}^n. \tag{4}$$

In the case of our downloaded `GoogleNews-vectors-negative300`, each record in the file contains one string and an associated 300-element vector, and the association between those strings and vectors defines $\mathbf{s}(w)$. The strings in the records define the domain W of $\mathbf{s}(w)$ and the vectors are the values. The order of records is not important to the definition of the function, but they are partially ordered by frequency of words in the corpus.

This is convenient for editing the function to omit rare words, as we do. Our function $\mathbf{s}()$ can thus be represented a mapping from words to an index, and a $150000 \times n$ array $i\mathbf{D}$. We can create a 'transformed' space $\mathbf{T}$ with m dimensions by building a function $\mathbf{t}(w) : W \to \mathbb{R}^m$, using the same word mapping as for $\mathbf{s}()$ and a new dictionary array, where C is an $n \times m$ array. We then call $\mathbf{T}$ a linear subspace of $\mathbf{S}$:

$$\mathbf{D}' = D \times C. \tag{5}$$

3.3 Evaluating analogies

An important function for numerical computation of word analogies is

$$\text{neighbors}_{\mathbf{S}}(v, n, m) : \mathbb{R}, \mathbb{N}, (\mathbb{R}^n, \mathbb{R}^n \to \mathbb{R}) \to W^n, \tag{6}$$

where v is a point in $\mathbb{R}^n$, n is a small integer, and m is a metric function. The $\text{neighbors}_{\mathbf{S}}()$ function obviously needs access to the dictionary function $\mathbf{s}$ in order to find a word from a point. The function $\text{neighbors}_{\mathbf{S}}(v, n, m)$ returns an ordered list L of n words $w_i \in W$ such that $\mathbf{s}(w_i)$ are closest to v according to m, that is

$$\forall(x \in W)\forall(w_i \in L)(m(x, v) < m(w_i, v)) \implies x \in L. \tag{7}$$

We denote pairs in a relation as p_i, and the two ordered elements of the pair as p_i^0 and p_i^1. Each pair has a difference vector d_i:

$$d_i = \left(\mathbf{s}(p_i^1) - \mathbf{s}(p_i^0)\right). \tag{8}$$

To compute the value of ? in in the analogy

$$p_i^0 : p_i^1 :: p_j^0 : X, \tag{9}$$

we evaluate

$$\text{neighbors}_{\mathbf{S}}(\left(d_i + s(p_j^0)\right), 1, \text{cosd}) = p_j^1, \tag{10}$$

where cosd is the cosine distance between two vectors. We hope to find that $X = p_j^1$.

3.4 Finding a Transform

Our hypothesis is that for each different relation there is a non-zero matrix $\mathbf{C}$, such that for any two pairs in the relation:

$$\left(\mathbf{s}(p_i^1) - \mathbf{s}(p_i^0)\right) \times \mathbf{C} \approx \left(\mathbf{s}(p_j^1) - \mathbf{s}(p_j^0)\right) \times \mathbf{C}, \tag{11}$$

that is

$$d_i \times \mathbf{C} \approx d_j \times \mathbf{C}. \tag{12}$$

In testing, we quantify $\approx$ in equation 11 and 12 to mean that the accuracy of solving the analogy (in the space $\mathbf{T}$ which is $\mathbf{S}$ transformed by $\mathbf{C}$) correctly is some fraction close to one. That is, we judge our success in finding C by the fraction of instances (i, j) in the relation which which satisfy Equation 10, but using the $\mathbf{T}$ space, instead of the $\mathbf{S}$ space.

We are working with 300-dimensional vectors, and we have from 19 to 40 pairs in each relation. We need to hold out at least one pair for testing; we experimented with holding out 2,4,6,8,10,14, and 16 pairs. The rest of the pairs in the relation are the training set. We want to train the values in C so that the difference vectors between the words in the pairs in the training set are transformed by the matrix multiplication into the mean difference vector for the training set.

Before training we place the difference vectors for the pairs in the training set in a matrix $\mathbf{D}$ and their (identical) targets in a target matrix T. We experimented with using both difference vectors and word-vectors with modified locations for training, but difference vectors give better results. The problem with this, is that a perfectly good solution could project all of S along the relation mean, densely packing the vector with points irrelevant to the relation.

Ideally, word points in the vector space for words not in the relation would either stay put, or shift without condensing. To encourage this to happen, we tried a strategy of *pinning* difference vectors. We add *pinned* vectors to the $\mathbf{D}$ and $\mathbf{T}$ matrices. These are a number of difference vectors between words chosen at random, which we want not to change; that is, we assume that a randomly chosen word pair (w_i, w_j) is not a pair of the relation, or is a *negative example*. For these vectors, the rows in $\mathbf{D}$ and $\mathbf{T}$ are the same, $\mathbf{s}(w_i) - \mathbf{s}(w_j)$. One of the nice benefits of training difference vectors

instead of training the individual words of a pair, is that we can be more confident that two randomly chosen words are not part of the relation we are training for. In the case of syntactic relations like verb-past-tense, a randomly chosen word is quite likely to be a verb, and potentially part of the relation, even though it might not be part of the provided examples. We train the matrix C with gradient descent, a learning rate of 0.001, for 3000 iterations, with a regularization constant of 0.98. Then, after training,

$$\mathbf{T} \approx \mathbf{D} \times \mathbf{C}. \tag{13}$$

A final part of the training goal is a regularization step. The objective function consists of two terms:

1. The sum of squares of each coordinate in $(\mathbf{T} - \mathbf{D} \times \mathbf{C})$.
2. A regularization term ρ. Considered as part of the objective function, this relates to a constant times the sum of the squared elements of the $\mathbf{C}$ array.

However, all we need during training is the regularization constant, $\rho \leq 1$, and the partial derivative δ_1 with respect to the $\mathbf{C}$ array, which we compute as:

$$\delta_1 = (2 * (\mathbf{D} \times \mathbf{C}) - \mathbf{T})^\top) \times \mathbf{D}. \tag{14}$$

and apply at each training step as:

$$\mathbf{C}' = (\rho)\mathbf{C} - (\text{LearningRate})\delta_1. \tag{15}$$

3.5 Evaluating the Analogies

In previous work with analogies, we have used two different kinds of normalization, in which every vector in the space $\mathbf{S}$ is changed. *Zero-centering* first computes the mean of all vectors in the space, then subtracts the mean from every vector. As a result, the new mean of each vector coordinate is zero. This translation does not affect the angles between difference vectors in the space. However angles with the new origin are different than angles with the old origin were, and points which were on a single line extending from the origin no longer are.

Thus the nearest neighbors of points in space as computed using cosine distance may change (Euclidean distances would not change). *Unit normalization* computes the square root of the sum of the squares of the coordinates of a vector, that is, the Euclidean distance to the origin, and divides all the coordinates by this number, effectively moving all points to the surface of a 300-dimensional hyper-sphere at unit distance from the origin. This transformation does not effect cosine distance, and it enables us to compute the cosine distance without recomputing vector norms, but it does change difference vectors.

The vectors between points on the surface of the sphere now point off into empty space when applied to other starting points. Of course, for sufficiently short vectors, they might not point *far* off the surface; but the angle from a word to its nearest neighbor is typically near $\pi/4$. This would put the calculated answer approximation for word analogies $\sqrt{(2)} - 1$ off the surface of the hyper-sphere, changing Euclidean distances (but not their relative ranking) to nearby words.

Furthermore the notion of parallel vectors on the surface of the sphere works only for small neighborhoods, so it is not obvious how to make use of difference vectors, let alone retrain them. In spite of these concerns, using these normalizations has apparently helped the success rate of analogy evaluation in the past. It has been suggested that just as unit normalization of vectors in the Information Retrieval vector space model [10] compensates for long documents, unit normalization of word vectors compensates for word frequency.

But since in this study we are specifically concerned with difference vectors, and difference vectors are impacted by normalization (and in previous studies difference vectors are not the main factor in accuracy evaluations, as may be seen in Table 1) we elected not to normalize for these experiments. Furthermore, Mikolov's word-search policy for finding the last word in the analogy explicitly rejects returning any of the first three words. This disallows some kinds of analogies, for example

$$\text{Prince Harry} : \text{Queen Elizabeth} :: \text{Prince William} : ? \tag{16}$$

where the answer is one of the guiding words, but more importantly, Linzen [9] points out that if the right answer is closer to the guide words than the computed vector,the computation and the difference vector are irrelevant, and only word similarity is being tested. So we don't eliminate the other words in the analogy from consideration, and we consider the answer wrong if the nearest neighbor of the computed vector turns out to be one of the other words in the analogy, even if the correct answer is closer than any other word except for the guide words.

We do keep track of this situation, primarily to able to compare the baseline figures with those of other researchers. We call this situation a *spare*, after the play in bowling in which some pins remain after the first ball, but are all knocked down by the second ball.

4 Experimental Results

The bar charts in Figure 2 and Figure 3 illustrate that the transformation improves performance on the word analogies for all of the four possible choices of pairs chosen from the training set andthe held-out, untrained pairs. This suggests that the transformation might actually accomplish the goal of isolating the semantic component of the relation that it is trained on, including on words on which it is not trained. Because the relations are small, and the chart is drawn from a particular training run, we see quite dramatic small number effects; the base accuracies between the four groups differ by large percentages, as do the trained accuracies.

The capital-common-countries relation is one of the best-performing of the Google set, but our statistics are lower than others, because we don't give credit for *spares*. We have a base performance before training of 179 successes for 506 analogies, or 0.35. We also note 236 spares, which would give a total accuracy of 0.82, but the spares are a measure of word-similarity, not of success in the difference vector calculation. Table 2 shows experiments working out the improvements on the held-out set for various parameter values, in this case for pinned words instead of difference vectors. Both Figure 2 and Figure 3 use 8 pairs held out and 50 pinned difference vectors.

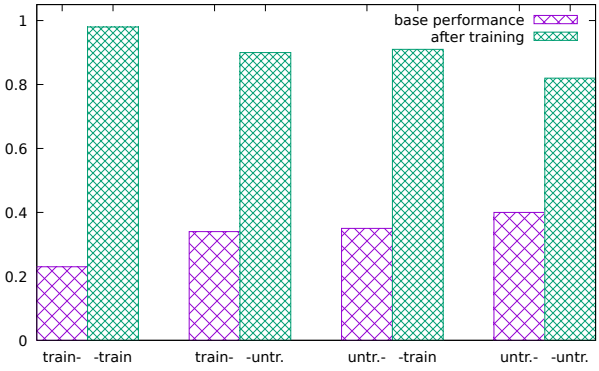


Fig. 2. Accuracy for capital-common-countries relation before and after trained transformation.

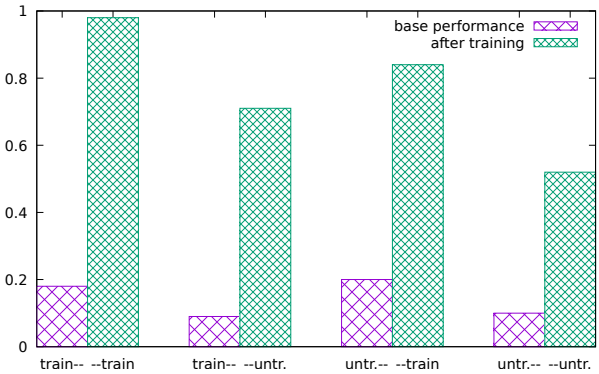


Fig. 3. Accuracy for `gram9-plural-verbs` relation before and after trained transformation.

The capital-common-countries relation has 23 pairs, and the held-out row labels at the left of the table show how many pairs were held out for testing. In the top row, four pairs are held out, so nineteen were used for training, and in the bottom row only seven pairs are available for training. The column headings show the number of pinned points, or negative examples, that are used in training. The best run in this table shows an absolute improvement of 0.16 in the fraction of correct results, while the surface plot shows the total accuracy. We built tables like Table 2 for all the relations, and then repeated the exercise focusing on the regions with the best results.

Table 3 shows the results of runs using the best set of parameters for each of the relations. In this table, the names of the relations are abbreviated to fit the table on a page, and several descriptive numbers are squeezed into the second column. The first two numbers are *pairs*, the number of pairs in the relation, and *analogies*, the number of analogies that can be built with the given number of pairs (always $pairs(pairs-1)$). The last two numbers in the second column are the training parameters: *held* is the number of pairs held out for testing, so

$$train = pairs - held. \tag{17}$$

Table 2. Improvements in analogy performance between untrained pairs in the capital-common-countries relation after training.

	0	**10**	**50**	**100**	**200**	**500**
4	0.00	0.00	0.08	0.08	0.08	0.00
6	0.07	0.10	0.13	0.13	0.13	0.03
8	0.02	0.07	0.14	0.16	0.09	0.05
10	0.01	0.09	0.10	0.13	0.10	0.04
12	0.00	0.05	0.12	0.10	0.07	0.04
14	-0.04	0.02	0.09	0.11	0.08	0.03
16	-0.05	0.03	0.11	0.13	0.09	0.05

Table 3. Some selected results for the google relations. tp=trained pair; hp = held-out pair.

Relation	pairs/analogies/ held/pinned	base accuracy	base accuracy with spares	trnd accuracy	tp: :hp base	tp: :hp trnd	tp: :hp base	tp: :hp trnd	hp: :tp base	hp: :tp trnd	hp: :tp base	hp: :tp trnd
capital-common...	23/506/14/150	35%	82%	89%	23%	98%	34%	90%	35%	91%	40%	82%
capital-world	39/1482/4/150	25%	76%	97%	25%	99%	30%	89%	23%	91%	25%	66%
city-in-state	40/1560/4/200	19%	79%	96%	20%	98%	10%	88%	18%	95%	0%	75%
country-curren...	19/342/6/100	13%	45%	64%	13%	92%	12%	44%	16%	48%	3%	13%
family	23/506/12/50	34%	84%	71%	56%	99%	31%	70%	34%	74%	20%	46%
gram1-adjectiv...	32/992/8/100	1%	29%	71%	0%	91%	2%	49%	2%	50%	5%	19%
gram2-opposite	28/756/6/100	1%	41%	84%	1%	95%	3%	53%	2%	83%	6%	50%
gram3-comparat...	37/1332/6/200	37%	89%	97%	42%	98%	31%	92%	23%	97%	13%	93%
gram4-superlat...	34/1122/10/150	20%	83%	92%	17%	99%	24%	88%	22%	92%	27%	58%
gram5-present-...	33/1056/10/50	6%	79%	83%	6%	99%	5%	68%	5%	78%	8%	48%
gram6-national...	39/1482/16/150	78%	92%	96%	80%	96%	73%	97%	80%	96%	79%	95%
gram7-past-ten...	40/1560/10/50	10%	66%	85%	10%	95%	9%	62%	11%	87%	11%	53%
gram8-plural	37/1332/4/50	5%	91%	97%	5%	99%	1%	84%	9%	97%	0%	75%
gram9-plural-v...	30/870/12/100	15%	68%	81%	18%	98%	9%	71%	20%	84%	10%	52%

Is the number of difference vectors from the relation in the training set; and *pinned* is the number of other difference vectors added to training set to stabilize it. Columns three and four of Table 3 also appear in Table 1. Column three, *base accuracy* is the fraction of analogies in the relation which are correctly solved with vector arithmetic, with our conditions: The vocabulary is restricted to only 150000 words, so that the number of words within any radius of the computed point is reduced, increasing slightly the likelihood that the nearest one is the given solution; and we insist that the word nearest the computed point is the only one considered (no spares).

Column four compares the accuracy when spares are allowed, thus partially answering the question "How could anyone think that a relation with a 1% accuracy rate is semantically interesting?" Column five is also a partial answer to that last question. It shows the accuracy after training and for every relation except `family`, it is higher than the traditional figure in column four, suggesting that with a little squeezing you can find some common semantics even if the concentration was not originally very high.

We can use *train* and *held* to compute the size of the four sub-relations described in the last eight columns of Table 3:

– **tp::tp** consists of word analogies constructed between pairs both of whose

difference vectors were in the training set. The size of this sub-relation is $train(train - 1)$.

- **tp::hp** consists of word analogies constructed with a pair from the training set on the left-hand side, and a pair from the held-out set on the right-hand side. The size of this sub-relation is $train(held)$.
- **hp::tp** consists of word analogies constructed with a pair from the held-out set on the left-hand side, and a pair from the training set on the right-hand side. The size of this sub-relation is also $train(held)$.
- **hp::hp** consists of word analogies constructed only with pairs from the held-out set. The size of this sub-relation is $held(held - 1)$.

For each of these sub-relations we show the base performance, that is, accuracy before training, and the trained performance. It is not a surprising result that after training, the performance of analogies from pairs in the training set (column 7, **tp::tp**, **trnd**) is very good. The lowest result is for the `gram1-adjective-adverb` relation, 91%.

However, it is interesting the sub-relations with mixed pairs from the trained and held-out sets (columns 9 and 11) also perform well; the lowest performance is for the `country-currency` relation, 44%, slightly lower than `gram1-adjective-adverb`, 49%. The two sub-relations seem to have similar performance, with many results clustered around 90%. The most interesting columns are the last two, in which we see that the accuracy of relations constructed with the held-out pairs has increased in every case.

The average accuracy rises from 18% in column 12 to 59% in column 12. These results seem to show that it is possible to build a linear transformation which isolates some of the semantic information whichpt a set of analogies depends upon. Two relations seem to generalize to untrained pairs particularly poorly, `country-currency` and `gram1-adjective-adverb`. Both also have very low base accuracy, and it seems possible that the relations just don't work; either the information is not in the semantic space, or it is stored in such a way that a linear transformation cannot enhance it.

Both seem to have little room for human error in building the relations – although perhaps currency is in flux, with the still on-going adoption of the Euro. An additional point raised, in particular by the contrast between base accuracy and the much larger accuracy with spares, is to what extent the Google relations include spurious pairs, not just from the point of view of additive semantics, but in terms of their similarities to each other. For example, the analogy

$$\text{man} : \text{woman} :: \text{King} : \text{Queen}. \tag{18}$$

Is solved by Mikolov's technique because 'King' and 'Queen' are very similar words in English, perhaps as a result of the long reigns of the two Queens Elisabeth and Queen Victoria.

The hypothetical gender vector, which should solve the `family` analogies, does not occur between 'King' and 'Queen'. 'Man' and 'woman' are also nearest neighbors, so the computed neighborhood of the answer is near 'King'; 'Queen' turns out to be the nearest word except for 'King'.

5 Discussion and Further Work

We have found a relation-specific linear transformation which improves the evaluation of word-analogies with vector arithmetic, including those pairs in the same relation which were held out of the training set. We believe that we are the first to consider this particular approach. We think that the approach works because the trained transformation discards information which is not common to the various pairs, while amplifying information which is. However, except for analogy performance, we have no evidence of this.

It seems possible that the analogy relations could be refined so that more of the pairs actually showed the same relationship; one way to do this might be to track the performance of individual pairs; those which consistently perform poorly in the held-out set may be candidate for removal. If word analogies can be enhanced to be more reliable, as our work suggests, then they may be good tools for obtaining semantic, morphological, or lexical information from semantic spaces. An advantage of linear transformations should be that it is easier to interpret how they interact with the data, compared to e.g. neural networks.

It might be that psychological analysis of meaning axes, as considered by Hollis and Westbury [5] could make use of this tool. A number of the parameters of this experiment were chosen arbitrarily. For example, refraining from normalization, while plausible, may not be necessary, and normalization may in fact have the salutary effects reported in other work. Other strategies to keep the vector space from condensing may work better than negative examples. The parameter space deserves to be explored more fully.

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